Simulation of coalescence and transport of bubbles and drops

Petr Karnakov, Sergey Litvinov and Petros Koumoutsakos*

ETH Zurich, Chair of Computational Science and Engineering

Clausiusstr. 33, 8092 Zurich, Switzerland

*petros@ethz.ch

Keywords: surface tension, volume of fluid, curvature, coalescence

Abstract

Numerical simulation of flows with bubbles and drops is challenging due to the presence of surface tension. Discretizations of surface tension rely on accurate estimation of curvature. The method of height-functions is commonly used for curvature estimation in the context of the volume-of-fluid approach. However, it requires evaluation of the height function on a stencil sufficient for calculation of the second derivative and therefore fails at low resolutions. A new method for curvature estimation is introduced in this work. The curvature is computed from a set of connected particles attracted to the piecewise linear interface. This gives values of curvature with relative error not exceeding 0.1 even at low resolutions up to one cell per radius. Various problems considered for verification and validation include: curvature of a sphere, translating droplet, near-wall coalescence of bubbles, coalescence of bubbles with satellite formation and inertial focusing of a bubble.

Introduction

Liquid jet atomization, boiling and bubble nucleation in chemical reactors are examples of applications where bubbles and drops of various scales are present simultaneously. Adaptive mesh refinement performs well in cases of isolated interfaces but not applicable if bubbles are distributed uniformly in the domain. For such cases, it is favorable that the numerical method resolves the features at large scales and ensures that at smaller scales the conservation laws are not violated. Among commonly used methods, both the level-set and the volume-of-fluid require at least five cells per radius of a spherical bubble.

This work introduces a new method for computing the curvature in the volume-of-fluid framework which allows for solving transport problems for bubbles and drops at low resolution up to one cell per radius. Small bubbles behave as tracer particles with conservation of mass and, as shown directly, correct translation by the flow.

Numerical Model

A two-component incompressible flow is described by the Navier-Stokes equations with surface tension and the advection equation for the volume fraction. A finite volume discretization is based on the SIMPLE method for pressure coupling (Ferziger 2012). The advection equation is solved using the volume-of-fluid method with picewise linear reconstruction (Aulisa 2007).

The curvature in two-dimensional space is computed for every cell from a set of particles attracted to the interface. The particles move until equilibration under constraints to maintain given distance and uniform angle between particles. This implies that they lie on a circle eventually giving the estimate of curvature. In three dimensions, same technique is applied to multiple cross-sections of the interface.

Verification and Validation

Curvature of sphere. Fig. 1 shows relative curvature error compared with generalized height-function method implemented in Gerris (Popinet 2009).
Translating droplet. Initial velocity is uniform in a periodic domain with $\mathrm{We} = 0.1$ and $\mathrm{La} = 1200$. Fig. 2 shows relative error in droplet velocity and evolution of the pressure jump relative to Laplace pressure.
Near-wall coalescence of bubbles. Coalescence of two bubbles placed at the wall. Bubble shapes compared to experiment (Soto 2018) in Fig. 3.
Coalescence of bubbles with satelite formation. Coalescence of two bubbles with formation of a smaller satellite bubble. Bubble shapes compared to experiment (Zhang 2008) in Fig. 4.
Inertial focusing of bubble in microchannel. Fig. 5 shows lateral position of the bubble of radius 0.24 of the channel height at $\mathrm{Re} = 7$ and $\mathrm{Ca} = 0.16$ compared to experiment (Hadikhani 2018)

Conclusions

The numerical model with a new technique for curvature estimation is validated against experimental results and at low resolutions shows significantly higher accuracy than the commonly used height-function method.

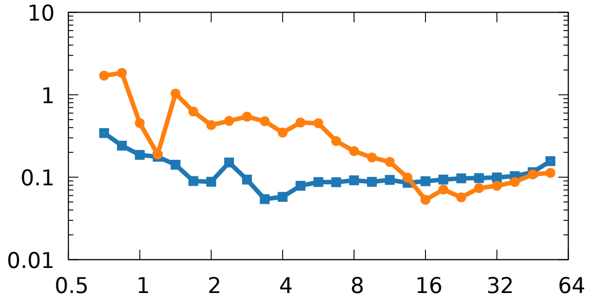

Figure 1: Curvature of a sphere. L_∞ norm of a relative error versus the number of cells per radius: present , Gerris .

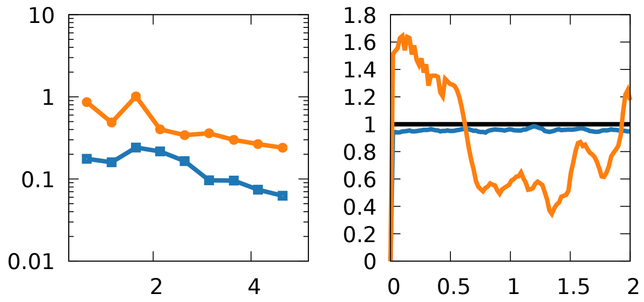

Figure 2: Translating droplet. Velocity error versus the number of cells per radius R/h (left) and evolution of the pressure jump for $R/h = 1.77$ (right): present , Gerris .

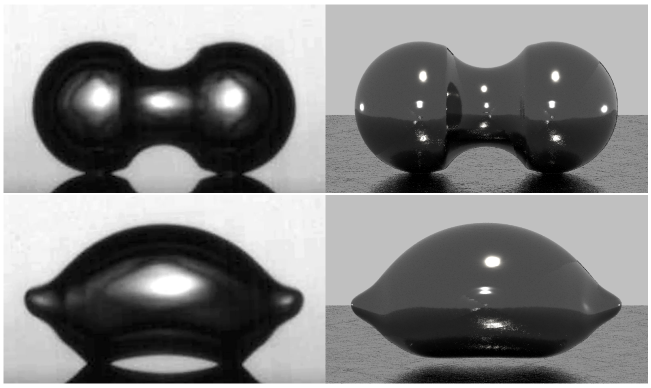

Figure 3: Near-wall coalescence of two bubbles: experiment (left), simulation (right).

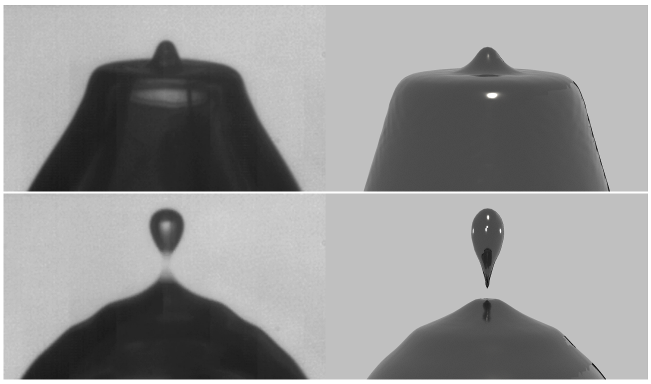

Figure 4: Coalescence of bubbles with satellite formation: experiment (left), simulation (right).

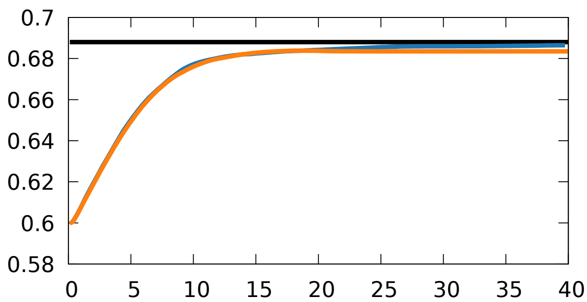

Figure 5: Evolution of lateral position of the bubble: experiment and simulation with 32 and 64 cells per channel height.

Acknowledgments

Experimental results for inertial focusing of bubbles in a microchannel provided by the Laboratory of Optics, EPFL.

References

Ferziger, J.H. and Peric, M. Computational methods for fluid dynamics. Springer Science & Business Media (2012)

Aulisa, E., Manservisi, S., et al. Interface reconstruction with least-squares fit and split advection in three-dimensional Cartesian geometry. Journal of Computational Physics, Vol. 225(2), pp. 2301 – 2319 (2007)

Popinet, S. An accurate adaptive solver for surface-tension-driven interfacial flows. Journal of Computational Physics, Vol. 228(16), pp. 5838 – 5866 (2009)

Soto, A.M., Maddalena, T., et al. Coalescence of diffusively growing gas bubbles. Journal of fluid mechanics, Vol. 846, pp. 143 – 165 (2018)

Zhang, F.H. & Thoroddsen, S.T. Satellite generation during bubble coalescence. Physics of Fluids, Vol. 20(2), p. 022104 (2008)

Hadikhani, P., Hashemi, S.M.H., et al. Inertial manipulation of bubbles in rectangular microfluidic channels. Lab on a Chip, Vol. 18(7), pp. 1035 - 1046 (2018)